Wind Power Plant Grid Integration for Future Power Systems

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Hochschule für Technik und Wirtschaft Berlin
University of Applied Sciences

Overview

1. Motivation

2. TS Framework (LPV) for Modeling, Estimation and Control

3. Wind Turbine Models for Controller Design

4. Controller Structure and Design for Power Tracking

5. Grid integration of wind turbines for frequency control

6. Conclusion

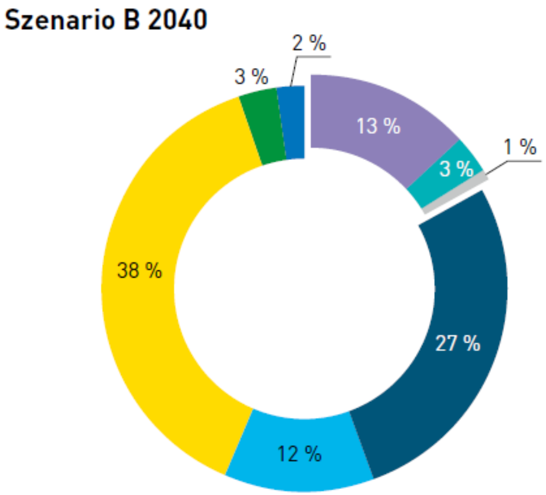
1. Motivation

Grid Integration of Wind Power Plants for Power Systems

- Share of renewable energies in total generation increases steadily

  Scenario 2040 for Germany with 327.1 GW installed capacity (today 224GW)

  PV (38%), Wind On-shore (27%), Wind Off-shore (12%)



- As a consequence
  - share of large concentrated rotating masses decreases
    - short circuit stability decreases
    - inertia decreases (low inertia system)
  - distributed systems substitute large concentrated power plants

Grid Integration of Wind Power Plants for Power Systems

- for stable grid operation renewable generators (REG) must work "like" power plants with bulk synchronous machines
- what "like" means is not yet clear until now

- in any case, the characteristics of REG must support
  - frequency control (instantaneous reserve, balancing energy)
  - voltage control (provision of reactive power, provision of short-circuit power, voltage regulation, power factor control
  - fault ride true
  - black-start capability

Grid Integration of Wind Power Plants for Power Systems

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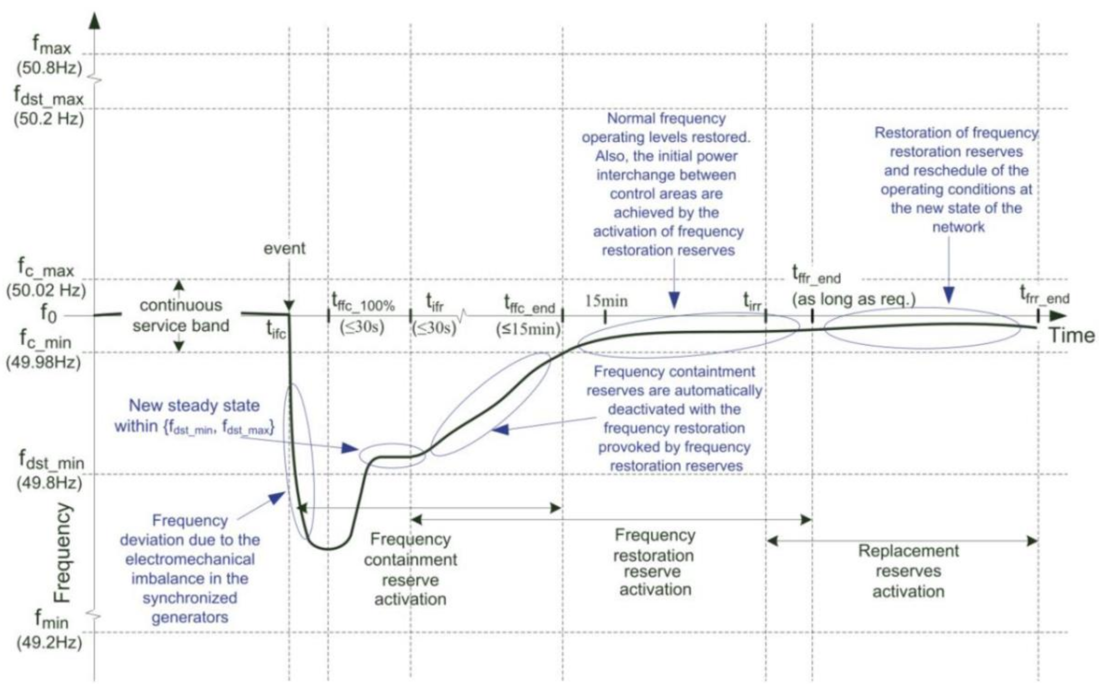
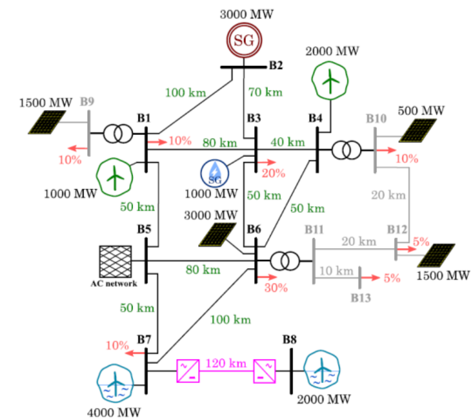
  - **<u>frequency control </u>**(instantaneous reserve, balancing energy)
  - voltage control (provision of reactive power, provision of short-circuit power, voltage regulation, power factor control
  - fault ride true
  - black-start capability
- Focus: Wind turbine control designs for frequency control

Grid Integration of Wind Power Plants for Power Systems

- Example of a frequency response

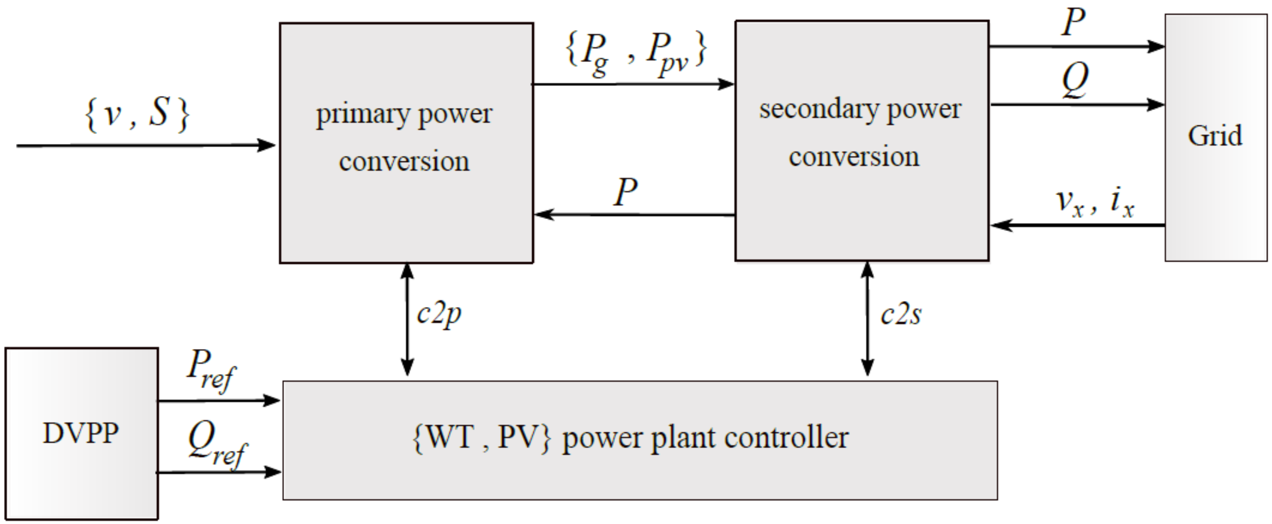




F. Díaz-González, M. Hau, A. Sumper and O. Gomis-Bellmunt, "Participation of wind power plants in system frequency control: Review of grid code requirements and control methods," Renewable and Sustainable Energy Reviews, vol. 34, pp. 551-564, 2014.

Grid Integration of Wind Power Plants for Power Systems

- General scheme of a renewable energy power plant



  - Primary power conversion: Wind energy in electrical energy (AC/DC)
  - Secondary power conversion: Grid side inverter
  - Power Plant Controller: Primary, secondary and power conditioning in between

Takagi-Sugeno Model (quasi LPV model)

$$\dot{\mathbf{x}} = \sum_{i=1}^{N_r} h_i(\mathbf{z}) \left(\mathbf{A}_i \, \mathbf{x} + \mathbf{B}_i \, \mathbf{u} \right), \qquad \mathbf{x}_0 = \mathbf{x}(t_0) \,,$$
$$\mathbf{y} = \sum_{i=1}^{N_r} h_i(\mathbf{z}) \, \mathbf{C}_i \, \mathbf{x}$$

weighted combinations of linear LTI systems

$$\dot{\mathbf{x}} = \sum_{i=1}^{N_r} h_i(\mathbf{z}) \left(\mathbf{A}_i \, \mathbf{x} + \mathbf{B}_i \, \mathbf{u} + \mathbf{a}_i \right), \qquad \mathbf{x}_0 = \mathbf{x}(t_0) \,,$$
$$\mathbf{y} = \sum_{i=1}^{N_r} h_i(\mathbf{z}) \left(\mathbf{C}_i \, \mathbf{x} + \mathbf{c}_i \right)$$

weighted combinations of affine systems

- choice of ***z*** vector allows very flexible use of the model

  - components $z_j \,, \quad j = 1, \ldots, l$ (premise variables) may include

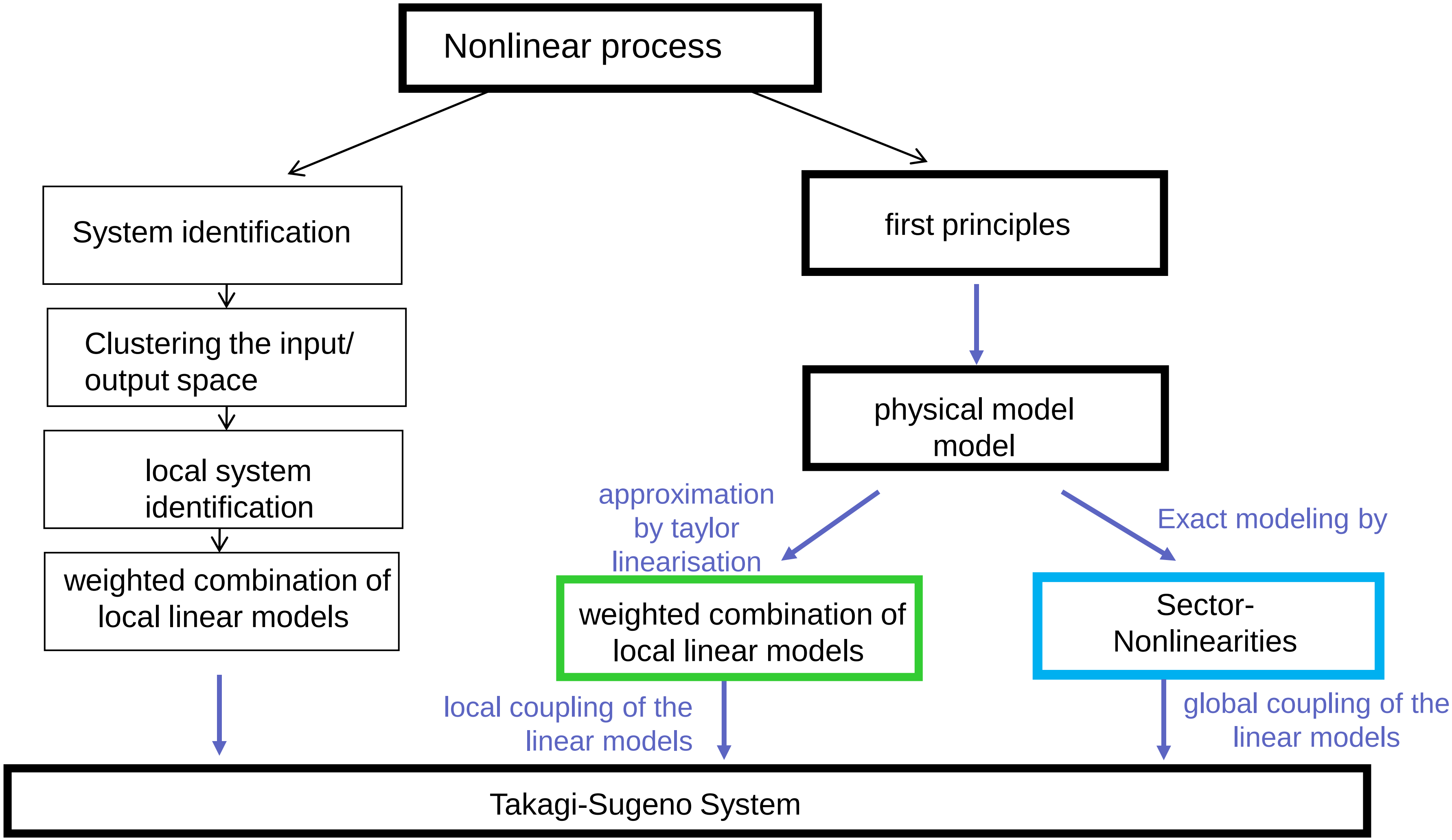
    a) measured states $x_i \,, \quad i = 1, \ldots, n$

    b) estimated states $\hat{x}_i \,, \quad i = 1, \ldots, n$

    c) system inputs $u_i \,, \quad i = 1, \ldots, m$ (not generally valid)

    d) variable parameters θ_i

2. TS Framework (LPV) for Modeling, Estimation and Control



2. TS Framework (LPV) for Modeling, Estimation and Control

approximation by taylor linearisation

weighted combination of local linear models

local coupling of the linear models

$$\dot{\mathbf{x}} = \sum_{i=1}^{N_r} h_i(\mathbf{z}) \left(\mathbf{A}_i\,\mathbf{x} + \mathbf{B}_i\,\mathbf{u} + \mathbf{a}_i\right), \qquad \mathbf{x}_0 = \mathbf{x}(t_0)\,,$$

$$\mathbf{y} = \sum_{i=1}^{N_r} h_i(\mathbf{z}) \left(\mathbf{C}_i\,\mathbf{x} + \mathbf{c}_i\right)$$

Publication #1: Florian Pöschke, Eckhard Gauterin, Martin Kühn, Jens Fortmann, Horst Schulte
Load mitigation and power tracking capability for wind turbines using LMI-based control design,
In ***Journal: Wind Energy,***
vol 23, issue 9, Sep. 2020, pp. 1792-1809,
https://doi.org/10.1002/we.2516

Publication #2: F. Pöschke, V. Petrović, F. Berger, L. Neuhaus, M. Hölling, M. Kühn, H. Schulte
Model-based wind turbine control design with power tracking capability: A wind-tunnel validation
In ***Control Engineering Practice 120 (2022) 105014***
https://doi.org/10.1016/j.conengprac.2021.105014

- Linerization is performed numerically with FAST (Fatigue, Aerodynamics, Structures, and Turbulence model)
- Advantage: Very dense grid of local models (240-260) covers an entire operating space
- Disadvantage: The model of the turbine must already be available with all data (airfoils etc. structural parameters)

2. TS Framework (LPV) for Modeling, Estimation and Control

Publication #3: Florian Pöschke, Horst Schulte
In: **Journal *at - Automatisierungstechnik, pp. 820-835, 2021, ISSN 0178-2312,***
https://doi.org/10.1515/auto-2021-0047

Exact modeling by

Sector-Nonlinearities

global coupling of the linear models

$$\dot{\mathbf{x}} = \sum_{i=1}^{N_r} h_i(\mathbf{z}) \left(\mathbf{A}_i\,\mathbf{x} + \mathbf{B}_i\,\mathbf{u}\right), \qquad \mathbf{x}_0 = \mathbf{x}(t_0)\,,$$

$$\mathbf{y} = \sum_{i=1}^{N_r} h_i(\mathbf{z})\,\mathbf{C}_i\,\mathbf{x}$$

- Sector Nonlinearities are derived analytically for different DOF models
- Advantage: exact representation of the nonlineare terms extracted via *hi(.)* functions
- Disadvantage: poor TS formulation can lead to conservative control design

2. TS Framework (LPV) for Modeling, Estimation and Control

Model-based Takagi-Sugeno Controller

- Integral-state feedback control law in TS form (Parallel distributed Compensator)

$$\mathbf{u} = \sum_{i=1}^{N_r} h_i(\mathbf{z}) \left(-\mathbf{K}_{x,i}\,\mathbf{x} + \mathbf{K}_{I,i}\,\mathbf{x}_I \right) \quad \text{where} \quad \mathbf{x}_I = \int_0^t \big(\mathbf{w}(\tau) - \mathbf{y}(\tau)\big) d\tau$$

- Observer in TS form (based on LTI and affine sub-systems)

$$\dot{\hat{\mathbf{x}}} = \sum_{i=1}^{N_r} h_i(\mathbf{z}) \left(\mathbf{A}_i\,\hat{\mathbf{x}} + \mathbf{B}_i\,\mathbf{u} + \mathbf{L}_i\,(\mathbf{y} - \hat{\mathbf{y}}) \right),$$

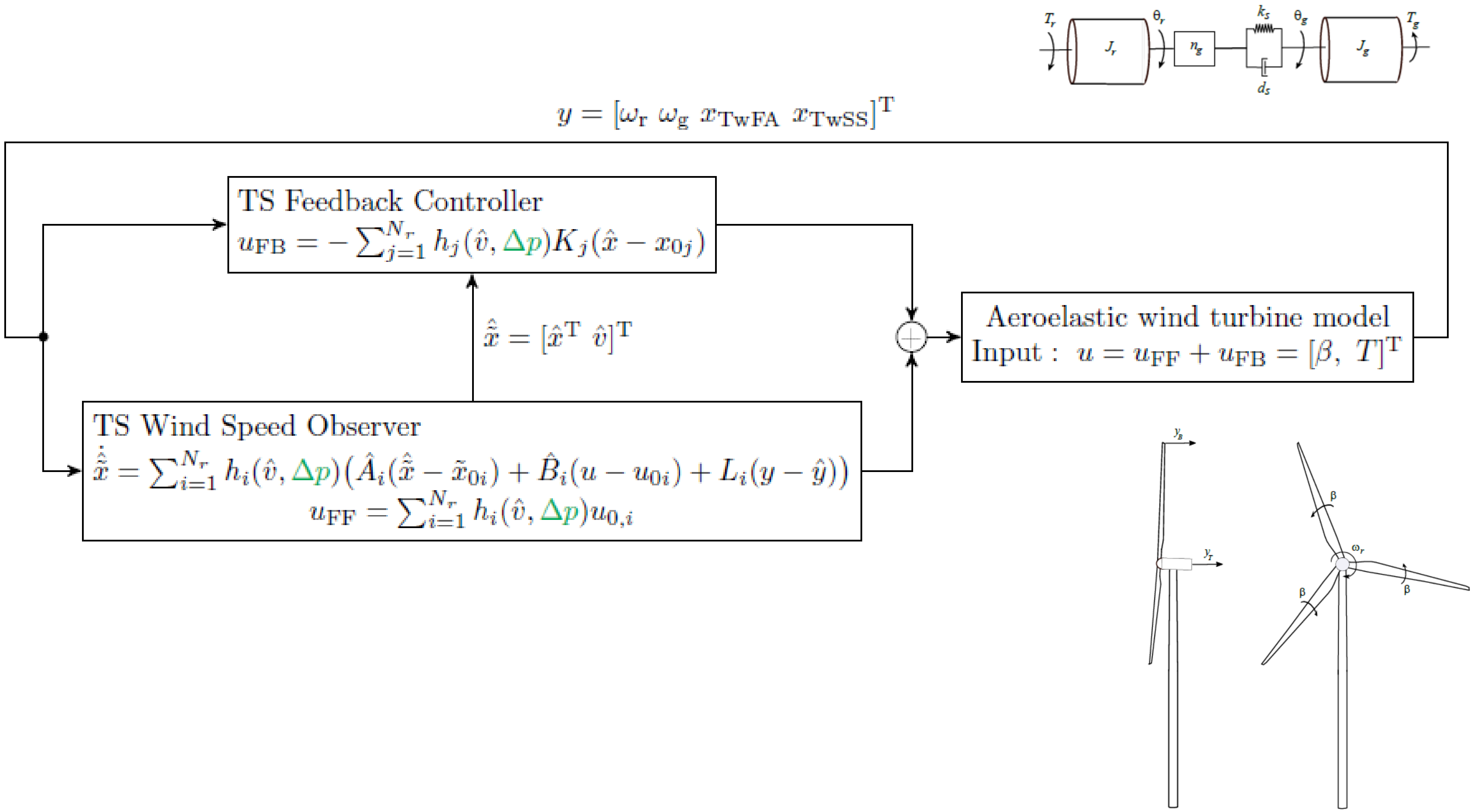
$$\hat{\mathbf{y}} = \sum_{i=1}^{N_r} h_i(\mathbf{z})\,\mathbf{C}_i\,\hat{\mathbf{x}}\,, \qquad \hat{\mathbf{x}}_0 = \hat{\mathbf{x}}(t_0)$$

$$\dot{\hat{\mathbf{x}}} = \sum_{i=1}^{N_r} h_i(\mathbf{z}) \left(\mathbf{A}_i\,\hat{\mathbf{x}} + \mathbf{B}_i\,\mathbf{u} + \mathbf{a}_i + \mathbf{L}_i\,(\mathbf{y} - \hat{\mathbf{y}}) \right),$$

$$\hat{\mathbf{y}} = \sum_{i=1}^{N_r} h_i(\mathbf{z}) \left(\mathbf{C}_i\,\hat{\mathbf{x}} + \mathbf{c}_i \right), \qquad \hat{\mathbf{x}}_0 = \hat{\mathbf{x}}(t_0)$$

observer is used for effective wind speed estimation (premise variables) and non measurable states

Example: wind turbine controller (primary side) for load mitigation and power tracking capability



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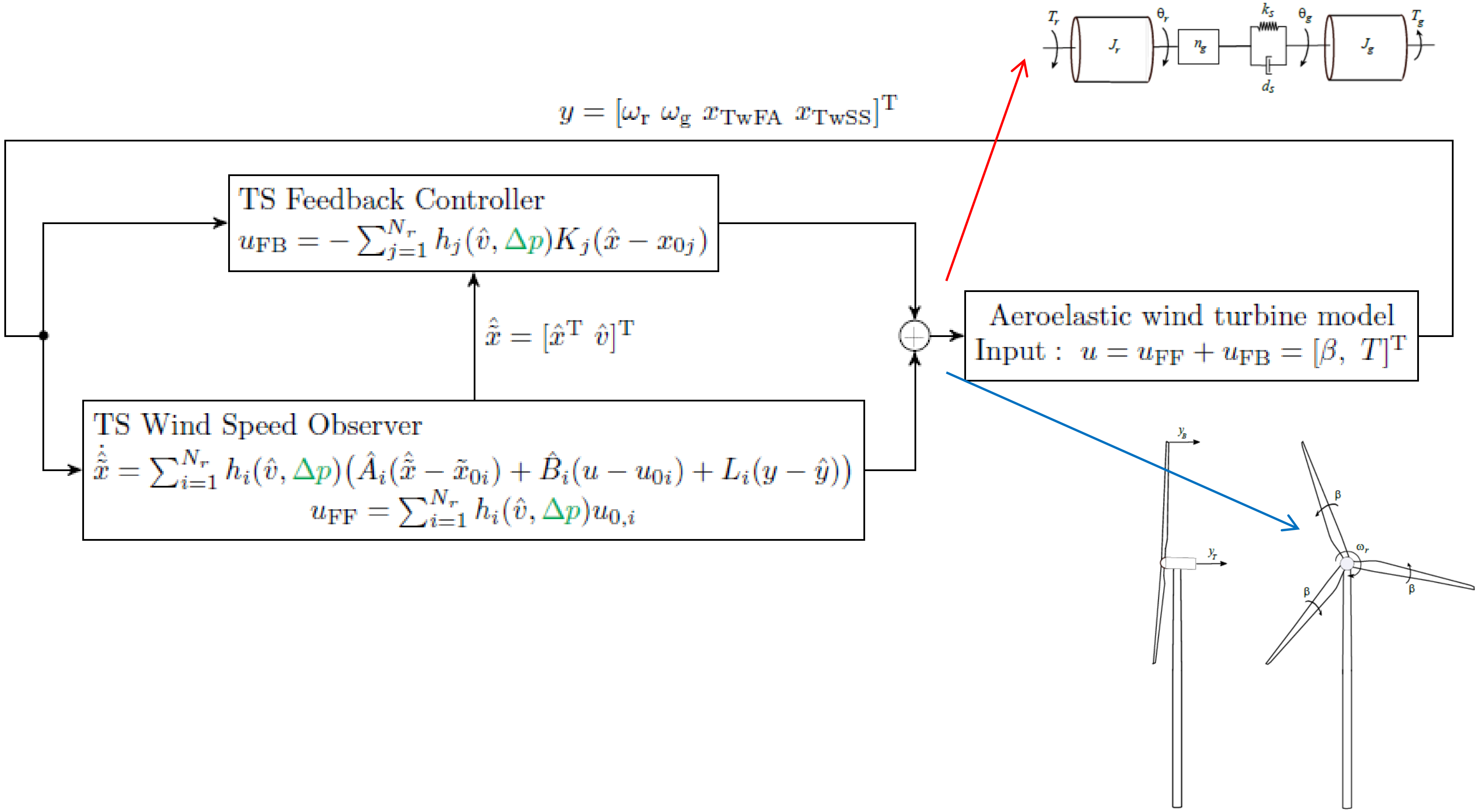
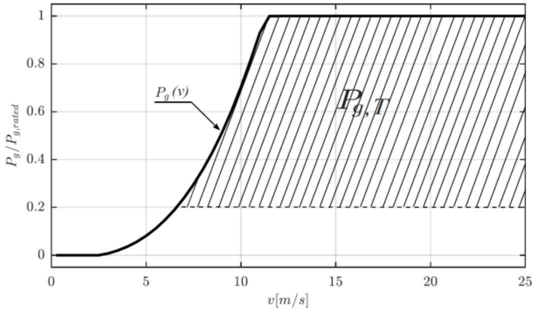


Control objectives

- Load mitigation for wind turbines
  - Load-mitigation operation (extreme/fatigue loads)
  - load reduction in blade roots, tower base, drive train
  - Reduction via pich and generator torque adjustment

- Power tracking capability for wind turbines (power frequency control)
  - Fast change of power generation
  - Negative and positive power reserve
  - Extension of the working range compared to the default control



$$P_{g,T} \in \{P_g \,|\, 0.2\, P_{g,rated} \leq P_g \leq P_g(v)\}$$

Control-oriented models related to the objectives

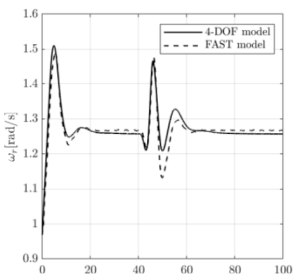
- 4-DOF model as initial model; from this, reduced models are derived

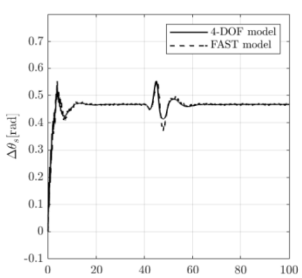
$$\dot{\mathbf{x}} = \underbrace{\begin{pmatrix} \mathbf{0}_{3\times3} & \tilde{\mathbf{A}}_{m_{12}} & \mathbf{0}_{3\times1} \\ -\mathbf{M}^{-1}\tilde{\mathbf{K}} & -\mathbf{M}^{-1}\mathbf{D} & \mathbf{0}_{4\times1} \\ \mathbf{0}_{1\times3} & \mathbf{0}_{1\times4} & -\frac{1}{\tau_\beta} \end{pmatrix}}_{\mathbf{A}} \mathbf{x} + \underbrace{\begin{pmatrix} \mathbf{0}_{4\times1} \\ \frac{1}{N\,m_B} F_T(\mathbf{x}, v) \\ \frac{1}{J_r} T_r(\mathbf{x}, v) \\ \mathbf{0}_{2\times1} \end{pmatrix}}_{\mathbf{f}(\mathbf{x},v)} + \underbrace{\begin{pmatrix} \mathbf{0}_{6\times1} & \mathbf{0}_{6\times1} \\ -\frac{1}{J_g} & 0 \\ 0 & \frac{1}{\tau_\beta} \end{pmatrix}}_{\mathbf{B}} \mathbf{u}$$

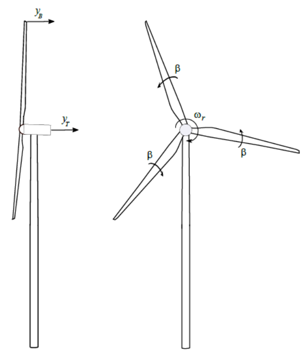
$$\mathbf{x} = \left(y_T\,,\, y_B\,,\, \Delta\theta_S\,,\, \dot{y}_T\,,\, \dot{y}_B\,,\, \omega_r\,,\, \omega_g\,,\, \beta\right)^T$$

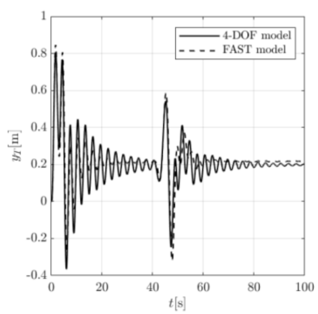
$$\mathbf{u} = \begin{pmatrix} T_g \\ \beta_{ref} \end{pmatrix}$$

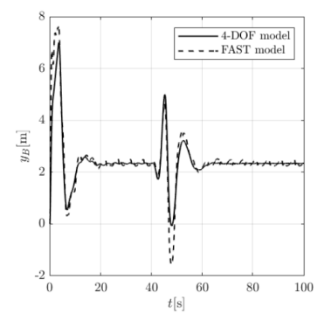
- Model validation with FAST (NREL)

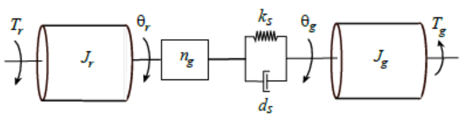












Control-oriented models related to the objectives

- 2-DOF for Load mitigation in the full-load region (v > vd)

$$\dot{\mathbf{x}} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_T}{m_T} & -\frac{d_T}{m_T} & f_3(\mathbf{x}, v) & f_4(\mathbf{x}, v) \\ 0 & 0 & f_1(\mathbf{x}, v) & f_2(\mathbf{x}, v) \\ 0 & 0 & 0 & -\frac{1}{\tau_\beta} \end{pmatrix} \mathbf{x} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{\tau_\beta} \end{pmatrix} u$$

$$\mathbf{x} = (\, y_T \,,\, \dot{y}_T \,,\, \omega_r \,,\, \beta \,)^T$$

$$u = \beta_{ref}$$

$$f_1(\mathbf{x}, v) = \frac{1}{2} \left(\frac{\rho \, \pi \, R^3 \, v^2 \, c_{Q,1}(\mathbf{x}, v)}{J} - \frac{n_g \, T_{g,rated}}{J} \right) \frac{1}{x_1}$$

$$f_2(\mathbf{x}, v) = \frac{\rho \, \pi \, R^3 \, v^2 \, c_{Q,2}(\mathbf{x}, v)}{2J} \, \frac{1}{x_2} \, .$$

$$f_3 = \frac{1}{2} \frac{\rho \, \pi \, R^3 \, v^2 \, C_{T,1}}{m_T} \frac{1}{x_3}$$

$$f_4 = \frac{1}{2} \frac{\rho \, \pi \, R^3 \, v^2 \, C_{T,2}}{m_T} \frac{1}{x_4}$$

- 1-DOF model for power tracking

$$\dot{\mathbf{x}} = \sum_{i=1}^{N_r=4} h_i(\mathbf{z}) \, \mathbf{A}_i \, \mathbf{x} + \mathbf{B} \, u \, , \qquad \mathbf{y} = \mathbf{C} \, \mathbf{x} \qquad \mathbf{z} = (\, \omega_r \,,\, \beta \,,\, v \,,\, T_g \,)^T$$

$$u = \beta_{ref}, \qquad \mathbf{x} = (x_1 \,,\, x_2)^T = (\omega_r \,,\, \beta)^T$$

4. Controller Structure and Design for power tracking

Power tracking: Basic principle

- power generated by the generator depends on speed and/or torque

$$P_g(t) = \omega_g(t) \cdot T_g(t)$$

- to avoid minimum phase behavior, either torque **or** speed must be changed
- power tracking capability is given by add-on feedforward control path
- two-control-region-approach (cross symmetrical)

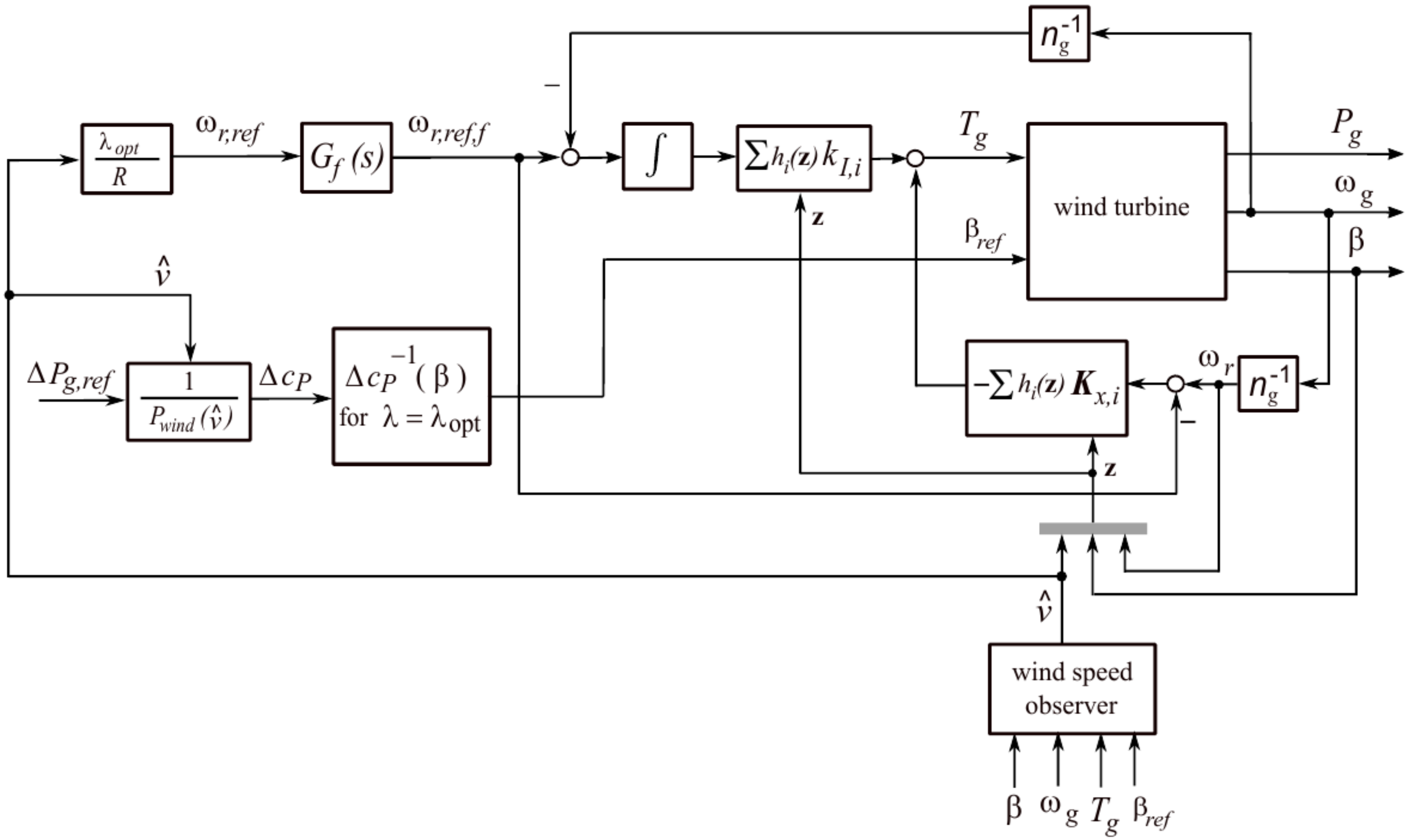
a) below rated (design) wind speed $v < v_d$

- rotor speed feedback control (tracking) for power optimization
- power tracking (below optimum) via pitch feedforward control

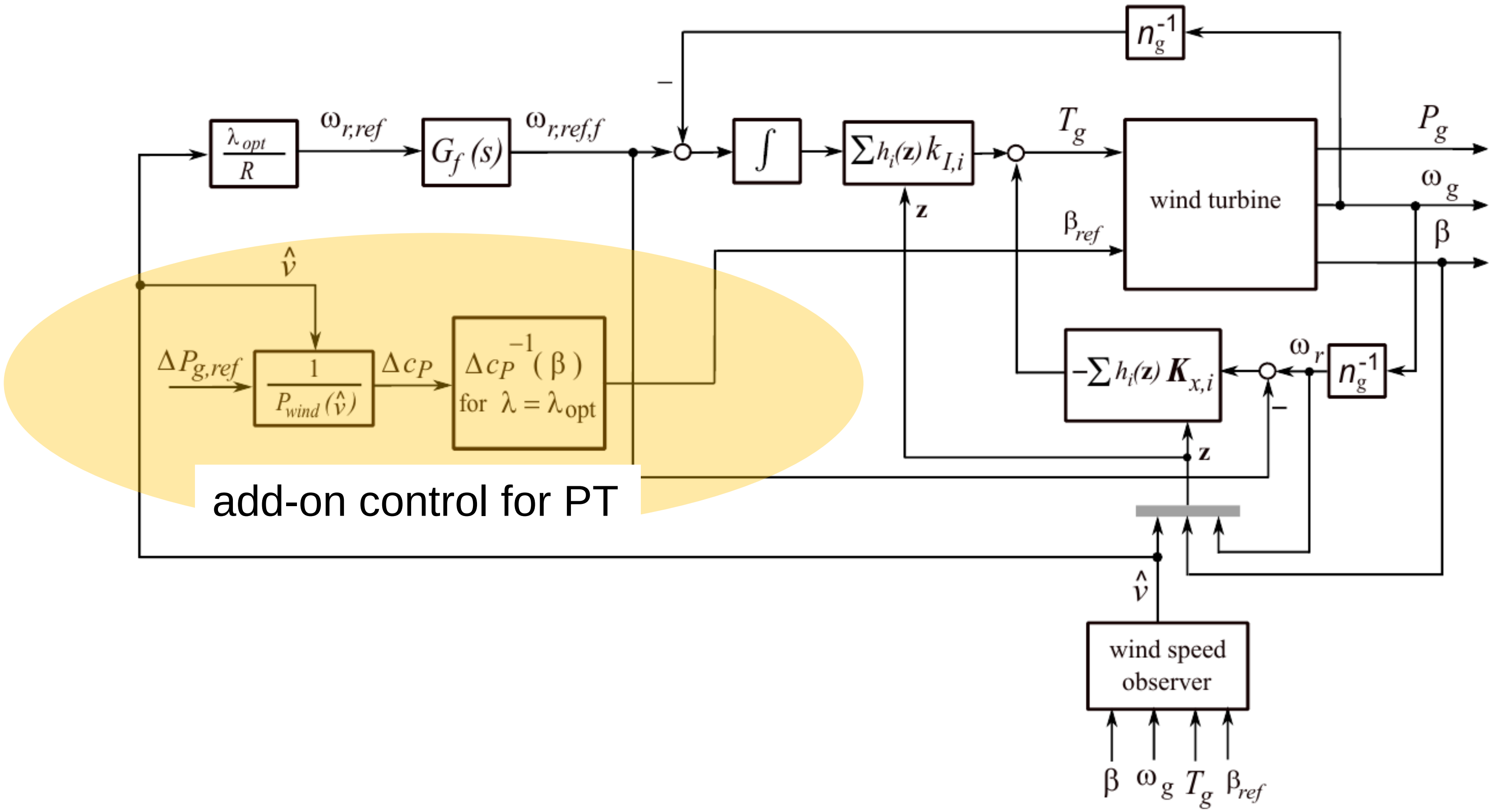
b) above rated (design) wind speed: $v \geq v_d$

- set point feedback control of generator/rotor at rated speed
- power tracking via generator torque adjustment (feedforward)

Control scheme (1) for power tracking below the rated wind speed

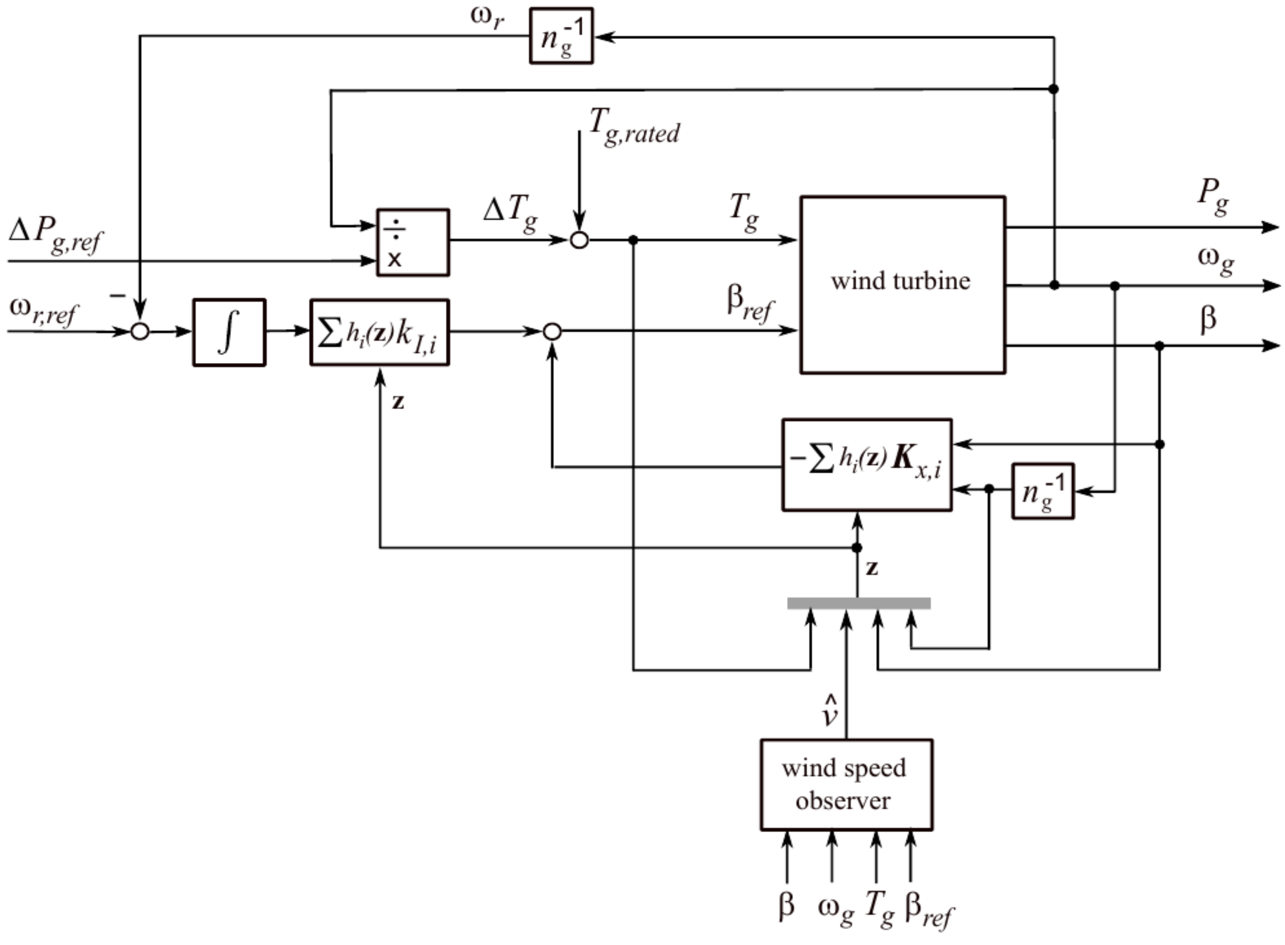


Control scheme (1) for power tracking below the rated wind speed

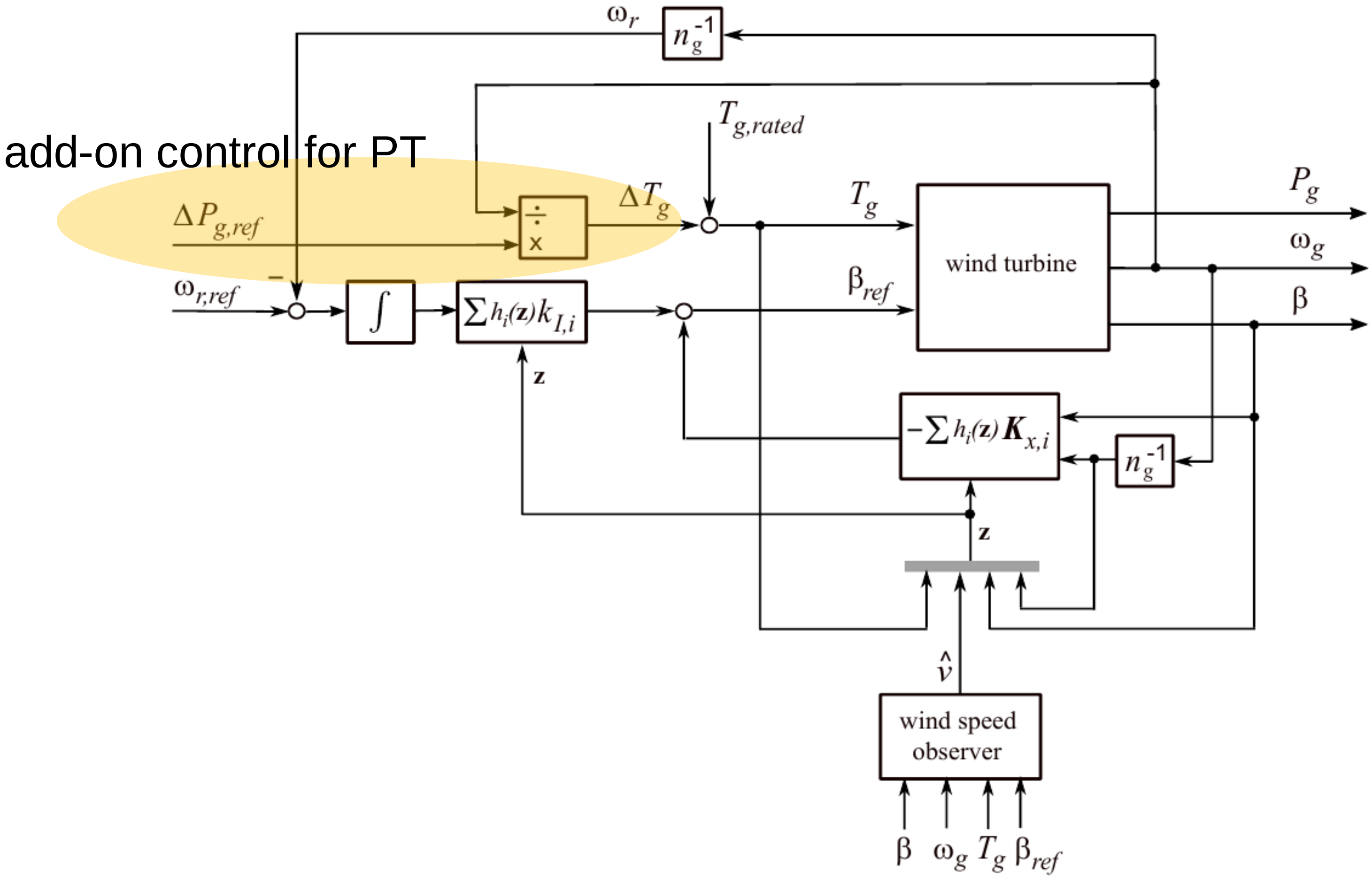


4. Controller Structure and Design for power tracking

Control scheme (2) for power tracking above the rated wind speed

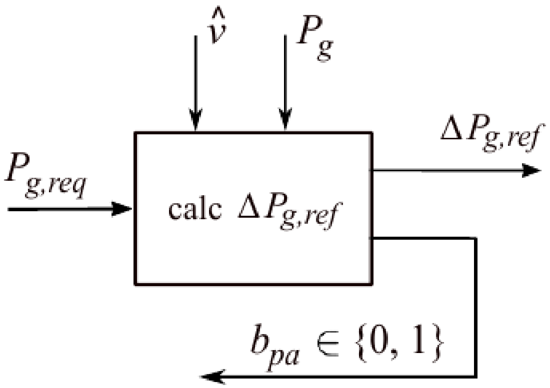


Control scheme (2) for power tracking above the rated wind speed



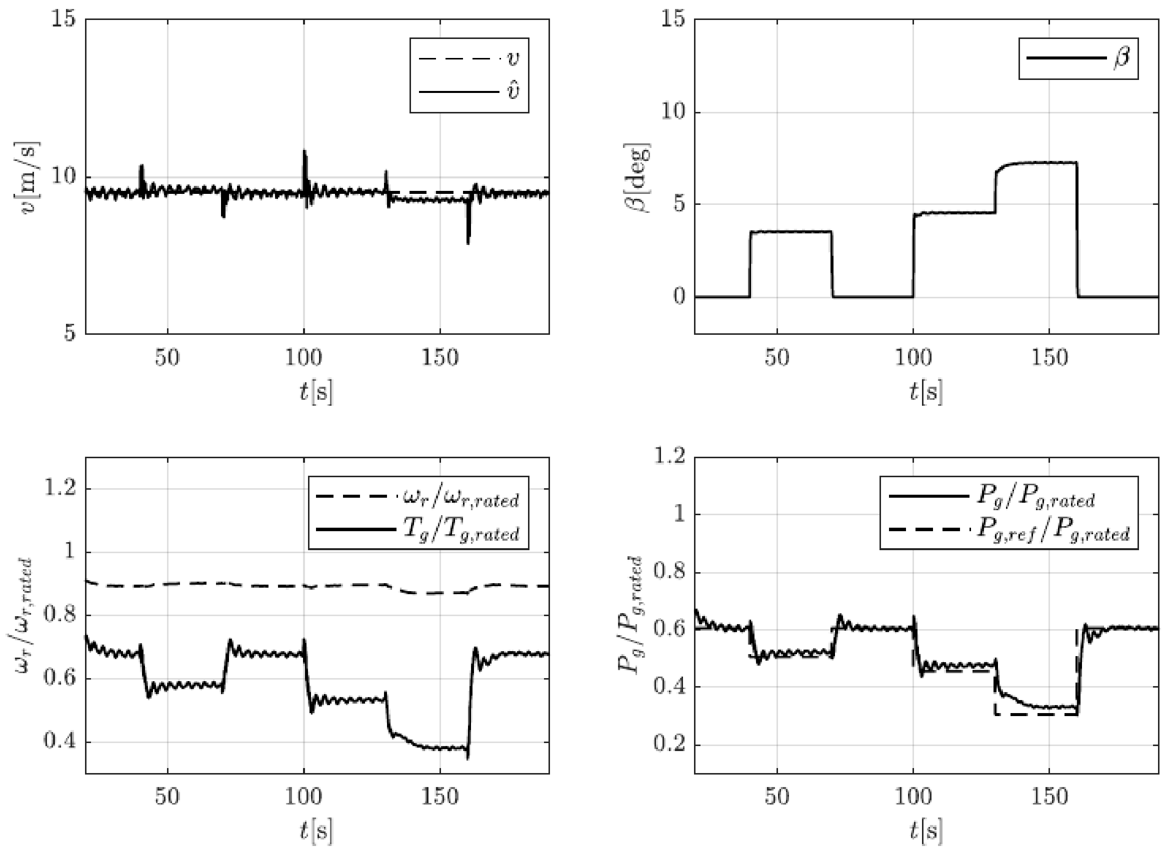
External power request (TSO controller) into reference value

$$\Delta P_{g,ref} = \begin{cases} 0 & , \quad P_{g,req} > P_{g,rated} \wedge \hat{v} > v_d \\ P_{g,req} - P_{g,rated} & , \quad P_{g,req} \leq P_{g,rated} \wedge \hat{v} > v_d \\ 0 & , \quad P_{g,req} > P_{g,av} \wedge \hat{v} \leq v_d \\ P_{g,req} - P_{g,av} & , \quad P_{g,req} \leq P_{g,av} \wedge \hat{v} \leq v_d \end{cases}$$

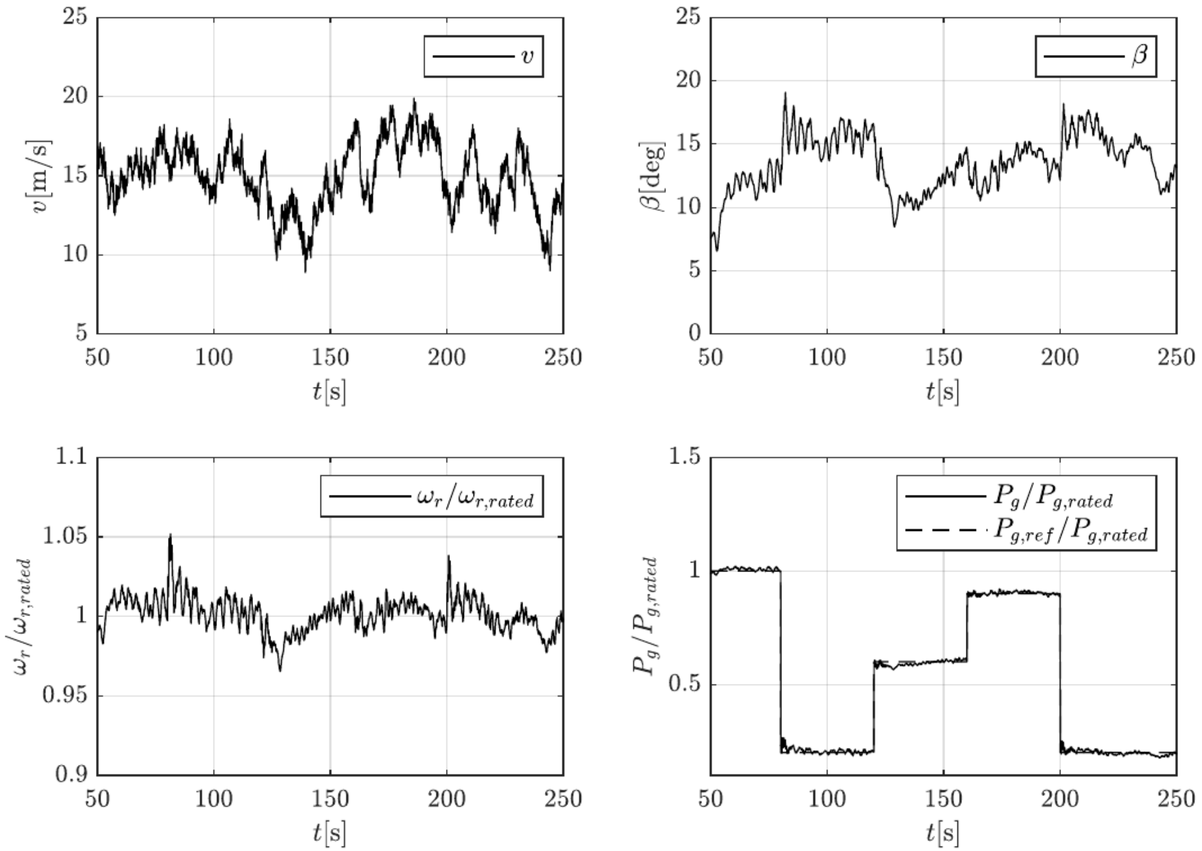


with the generated average power $$P_{g,av}(t) = \frac{1}{N} \sum_{i=0}^{N-1} P_g(t - i\,T_s)$$

Simulation result related to control scheme (1) $v < v_d$

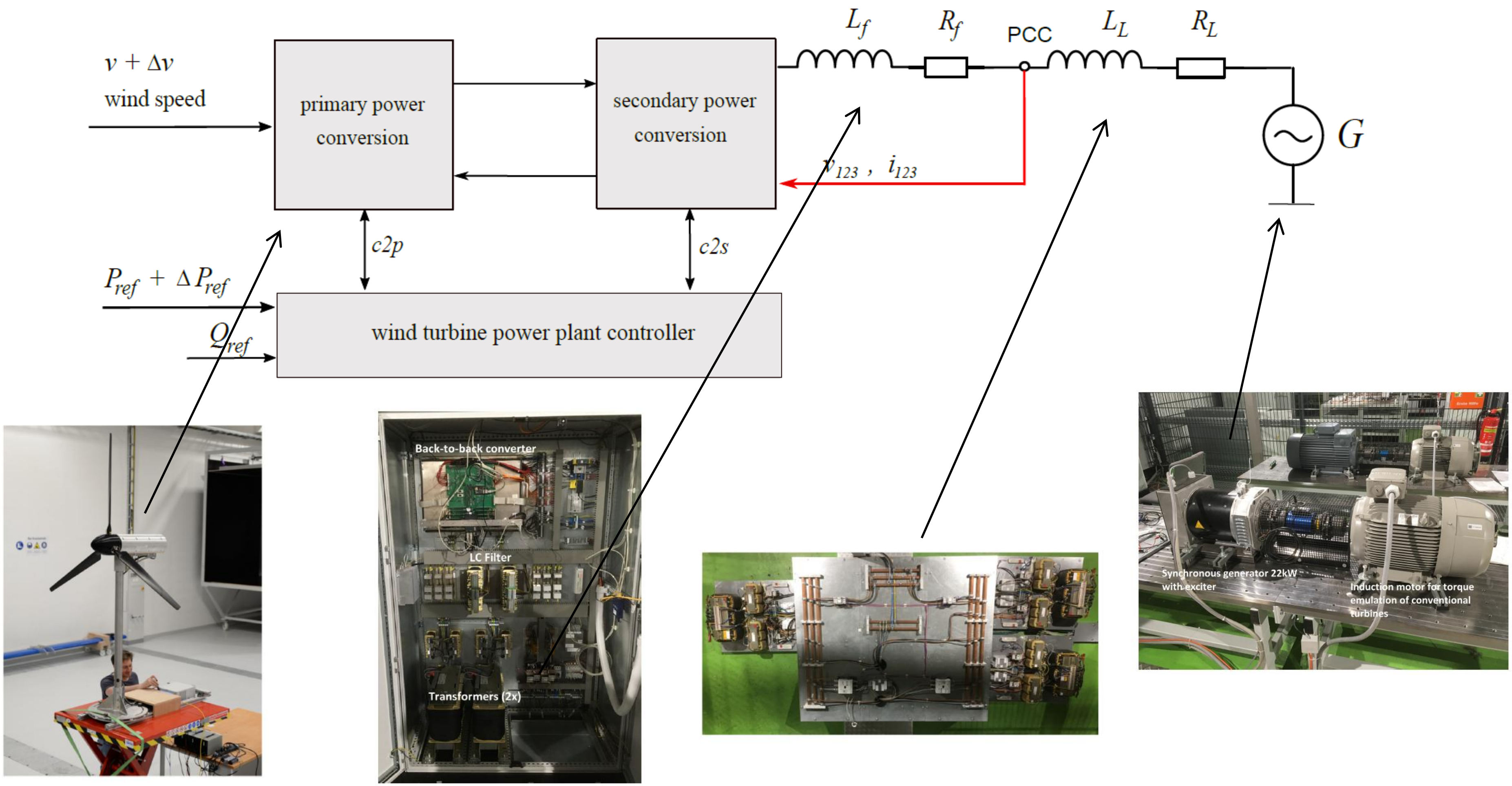


Simulation result related to control scheme (2) $v \geq v_d$

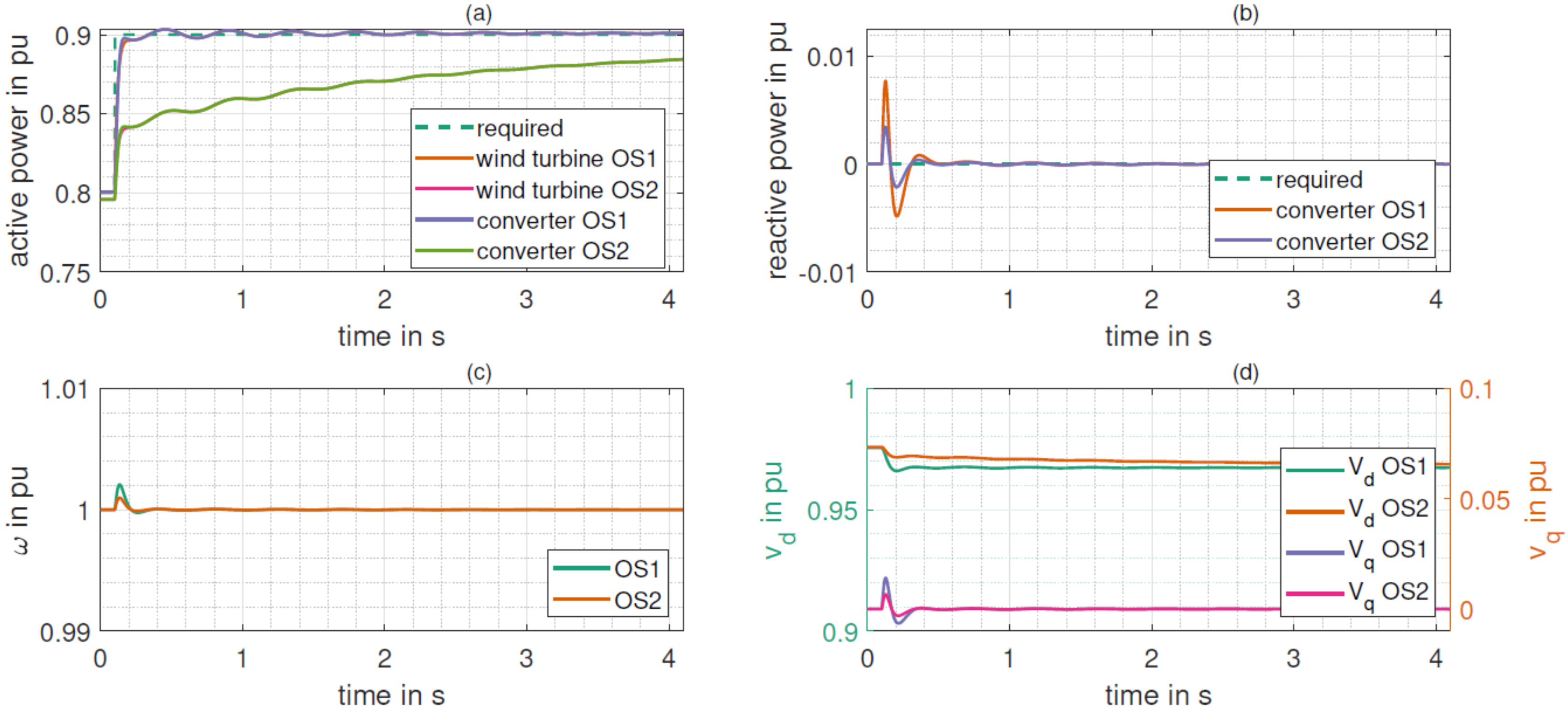


5. Grid integration of wind turbines for frequency control

Coupling of the Wind turbine power plant to the grid

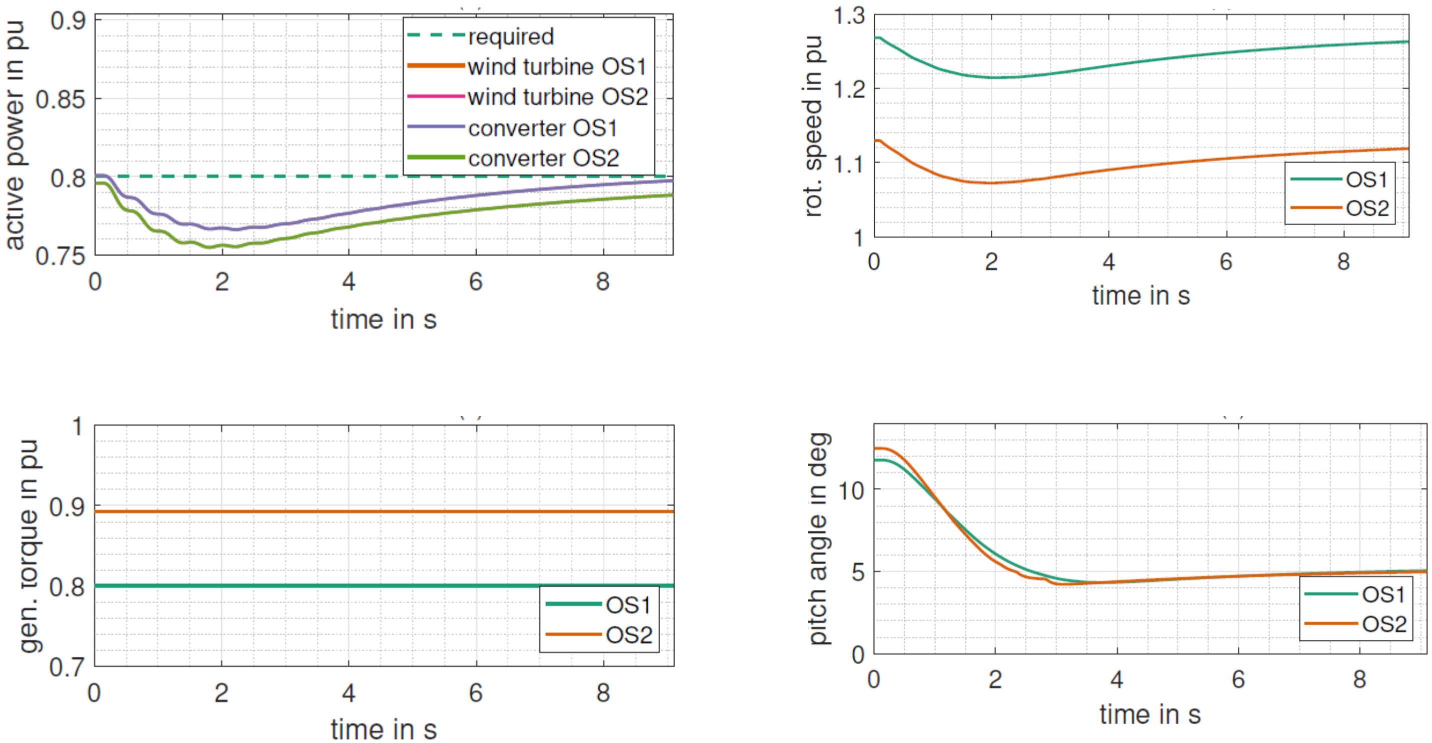


Case Study A: Power request $\Delta P_{req} = 0.1pu$



Wind power plant simulation results in test case 1: (a) active power (b) reactive power (c) estimated frequency of the phase locked-loop (d) voltage at the PCC

Case Study B: Decrease in wind inflow $\Delta v = -0.3pu$



6. Conclusion

Objective was the power tracking ability for wind turbines/farms

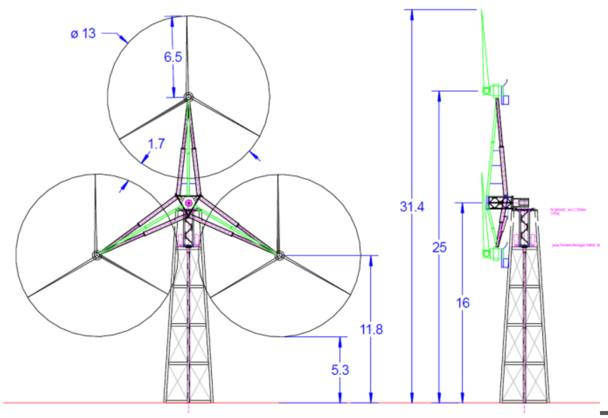
- Model based approach (TS models as class of nonlinear systems)
- Controller design could not be addressed in the workshop talk

  please contact me schulte@htw-berlin.de

- On the other hand, the controller structure and integration into a higher level power control system were presented in detail
- Controller was sucessfully implemented on a wind tunnel test-bed system
- Next step: Implementation on a 90kW

  multi-rotor system

  (demonstrator)



TS modeling methods

Approximate modeling

- Condition: continuously differentiable right side of dynamic system

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{f}(\boldsymbol{x}(t), \boldsymbol{u}(t)) \qquad \boldsymbol{x} \in \mathbb{R}^n, \boldsymbol{u} \in \mathbb{R}^m$$

$$\boldsymbol{y}(t) = \boldsymbol{g}(\boldsymbol{x}(t), \boldsymbol{u}(t)) \qquad \boldsymbol{y} \in \mathbb{R}^p$$

- Let be given a solution $(\boldsymbol{x}_s(t), \boldsymbol{y}_s(t))$ for $\boldsymbol{u}_s : [t_0, \infty) \rightarrow \mathbb{R}^m$

- then, according to Taylor, the right-hand sides can be developed in the neighborhood of this solution

$$\frac{d(\boldsymbol{x}_s + \Delta \boldsymbol{x})}{dt} = \boldsymbol{f}(\boldsymbol{x}_s + \Delta\boldsymbol{x}, \boldsymbol{u}_s + \Delta\boldsymbol{u})$$

$$= \boldsymbol{f}(\boldsymbol{x}_s, \boldsymbol{u}_s) + \boldsymbol{A}(t)\,\Delta\boldsymbol{x} + \boldsymbol{B}(t)\,\Delta\boldsymbol{u} + \boldsymbol{r}_f(\boldsymbol{x}_s, \boldsymbol{u}_s, \Delta\boldsymbol{x}, \Delta\boldsymbol{u})$$

Approximate modeling

- time-variable matrices ***A***(t) and ***B***(t) are determined by the functional matrices

$$\boldsymbol{A}(t) := \boldsymbol{A}(\boldsymbol{x}_s(t), \boldsymbol{u}_s(t)) = \left[\frac{\partial \boldsymbol{f}(\boldsymbol{x}_s(t), \boldsymbol{u}_s(t))}{\partial \boldsymbol{x}(t)}\right] ,$$

$$\boldsymbol{B}(t) := \boldsymbol{B}(\boldsymbol{x}_s(t), \boldsymbol{u}_s(t)) = \left[\frac{\partial \boldsymbol{f}(\boldsymbol{x}_s(t), \boldsymbol{u}_s(t))}{\partial \boldsymbol{u}(t)}\right]$$

along the solution (trajectory) $\boldsymbol{x}_s(t)$ for a given input $\boldsymbol{u}_s(t)$

- with neglection of the residual term $\boldsymbol{r}_f(\boldsymbol{x}_s, \boldsymbol{u}_s, \Delta\boldsymbol{x}, \Delta\boldsymbol{u})$, we get

$$\dot{\boldsymbol{\xi}}(t) = \boldsymbol{f}(\boldsymbol{x}_s(t), \boldsymbol{u}_s(t)) + \boldsymbol{A}(t)\ (\boldsymbol{\xi}(t) - \boldsymbol{x}_s(t)) + \boldsymbol{B}(t)\ (\boldsymbol{u}(t) - \boldsymbol{u}_s(t)) ,$$
$$\boldsymbol{\xi}(t_0) = \boldsymbol{x}_s(t_0) ,$$

Approximate modeling

- Now let be given a set of discrete points on the solution

$$\mathbb{G}_s = \{\boldsymbol{x}_i, \boldsymbol{u}_i\} \in (\boldsymbol{x}_s(t), \boldsymbol{u}_s(t)) \ , \qquad i = 1, \ldots, N_r \ , \qquad t \in [t_0, \infty) \subset \mathbb{R}$$

- piecewise approximation of the right-hand side of the previous nonlinear systems by a weighted combination of affine linear systems

$$\dot{\tilde{\boldsymbol{\xi}}}(t) = \sum_{i=1}^{N_r} \alpha_i(\tilde{\boldsymbol{\xi}}(t), \boldsymbol{u}(t)) \left[\boldsymbol{A}(\boldsymbol{x}_i, \boldsymbol{u}_i)\,(\tilde{\boldsymbol{\xi}}(t) - \boldsymbol{x}_i) + \boldsymbol{B}(\boldsymbol{x}_i, \boldsymbol{u}_i)\,(\boldsymbol{u}(t) - \boldsymbol{u}_i) + \boldsymbol{f}(\boldsymbol{x}_i, \boldsymbol{u}_i)\right]$$

$$\tilde{\boldsymbol{\xi}}(t_0) = \boldsymbol{x}_s(t_0)$$

- Summarizing the constant terms results in

$$\dot{\tilde{\boldsymbol{\xi}}}(t) = \sum_{i=1}^{N_r} \alpha_i(\tilde{\boldsymbol{\xi}}(t), \boldsymbol{u}(t)) \boldsymbol{A}(\boldsymbol{x}_i, \boldsymbol{u}_i)\,\tilde{\boldsymbol{\xi}}(t) + \sum^{N_r} \alpha_i(\tilde{\boldsymbol{\xi}}(t), \boldsymbol{u}(t)) \boldsymbol{B}(\boldsymbol{x}_i, \boldsymbol{u}_i)\,\boldsymbol{u}(t)$$
$$+ \sum_{i=1}^{N_r} \alpha_i(\tilde{\boldsymbol{\xi}}(t), \boldsymbol{u}(t)) \left[\boldsymbol{f}(\boldsymbol{x}_i, \boldsymbol{u}_i) - \boldsymbol{A}(\boldsymbol{x}_i, \boldsymbol{u}_i)\,\boldsymbol{x}_i - \boldsymbol{B}(\boldsymbol{x}_i, \boldsymbol{u}_i)\,\boldsymbol{u}_i\right]$$

Approximate modeling

- with the abbreviations $\boldsymbol{A}_i := \boldsymbol{A}(\boldsymbol{x}_i, \boldsymbol{u}_i)\,, \quad \boldsymbol{B}_i := \boldsymbol{B}(\boldsymbol{x}_i, \boldsymbol{u}_i)\,,$

$$\boldsymbol{a}_i := \boldsymbol{f}(\boldsymbol{x}_i, \boldsymbol{u}_i) - \boldsymbol{A}(\boldsymbol{x}_i, \boldsymbol{u}_i)\,\boldsymbol{x}_i - \boldsymbol{B}(\boldsymbol{x}_i, \boldsymbol{u}_i)\,\boldsymbol{u}_i$$

one obtains the T-S Form with an affine term (not included in the original work of Takagi/Sugeno)

$$\dot{\tilde{\boldsymbol{\xi}}}(t) = \sum_{i=1}^{N_r} \alpha_i(\tilde{\boldsymbol{\xi}}(t), \boldsymbol{u}(t))\,\boldsymbol{A}_i\,\tilde{\boldsymbol{\xi}}(t) + \sum_{i=1}^{N_r} \alpha_i(\tilde{\boldsymbol{\xi}}(t), \boldsymbol{u}(t))\,\boldsymbol{B}_i\,\boldsymbol{u}(t) + \sum_{i=1}^{N_r} \alpha_i(\tilde{\boldsymbol{\xi}}(t), \boldsymbol{u}(t))\,\boldsymbol{a}_i$$

where $\boldsymbol{z} = \begin{bmatrix} \tilde{\boldsymbol{\xi}}^T & \boldsymbol{u}^T \end{bmatrix}^T$

Exact modeling: Sector Nonlinearity Approach:

Pendulum $\ddot{\varphi} = -\frac{g}{l}\sin\varphi + \frac{1}{m\,l^2}M$ $\quad \mathbf{x} = (\varphi \quad \dot{\varphi})^T \quad u = M$

State-space form $\dot{\mathbf{x}} = \begin{pmatrix} 0 & 1 \\ -\frac{g}{l}\,\frac{\sin x_1}{x_1} & 0 \end{pmatrix}\mathbf{x} + \begin{pmatrix} 0 \\ \frac{1}{m\,l^2} \end{pmatrix} u = \mathbf{A}(\mathbf{x})\,\mathbf{x} + \mathbf{B}\,u$

Nonlinear function $f(x_1) = -\frac{g}{l}\,\frac{\sin x_1}{x_1} = w_1(x_1)\,\overline{f} + w_2(x_1)\,\underline{f}$

$\overline{f}, \underline{f}$: max/min values of f

Sector functions $w_1(x_1) := \frac{f(x_1) - \underline{f}}{\overline{f} - \underline{f}} \qquad w_2(x_1) := \frac{\overline{f} - f(x_1)}{\overline{f} - \underline{f}}$

Modeling with TS systems

Exact modeling: Sector Nonlinearity Approach:

Nonlinear system matrix can be written as

$$\begin{pmatrix} 0 & 1 \\ -\frac{g}{l}\,\frac{\sin x_1}{x_1} & 0 \end{pmatrix} = \begin{pmatrix} 0 & w_1 + w_2 \\ w_1\,\overline{f} + w_2\,\underline{f} & 0 \end{pmatrix} \quad w_1 + w_2 = 1$$

$$= w_1 \begin{pmatrix} 0 & 1 \\ \overline{f} & 0 \end{pmatrix} + w_2 \begin{pmatrix} 0 & 1 \\ \underline{f} & 0 \end{pmatrix} = w_1\,\mathbf{A}_1 + w_2\,\mathbf{A}_2$$

and the whole state-space model as

$$\dot{\mathbf{x}} = \begin{pmatrix} 0 & 1 \\ -\frac{g}{l}\,\frac{\sin x_1}{x_1} & 0 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 0 \\ \frac{1}{m\,l^2} \end{pmatrix} u = \sum_{i=1}^{N_r=2} w_i\,(x_1)\,(\mathbf{A}_i\mathbf{x} + \mathbf{B}\,u)$$

The Nonlinearity has been shifted from the system matrix to the membership functions.